EXTENDED NUMERICAL MODELING OF FATIGUE BEHAVIOR

THÈSE N$^{\underline{O}}$ 1617 (1997)

PRÉSENTEE AU DEPARTEMENT DE GENIE CIVIL

ÉCOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

POUR L'OBTENTION DU GRADE DE DOCTEUR ES SCIENCES TECHNIQUES

PAR

EERIK PEEKER

Ingénieur civil diplômé UTT
originaire de Tallinn (Estonie)

acceptée sur proposition du jury :

Prof. M.A. Hirt, rapporteur
Prof. J. Botsis, corapporteur
Prof. C. Castiglioni, corapporteur
Dr. V. Dubois, corapporteur
Dr. V. Esslinger, corapporteur
Dr. P. Kunz, corapporteur

Lausanne, EPFL
1997

Gilles Deleuze, philosopher : « *Fatigue - c'est la formulation biologique de ce que la journée est finie : j'ai fait ce que j'ai pu aujourd'hui. On ne tirera plus rien de moi. ... C'est une coda, la fatigue ...* ».

SUMMARY

The fatigue evaluation of existing structures is becoming more and more important. However, experimental study of the fatigue behavior of existing structures is expensive, the use of *S-N* curves produces results which are not precise enough, and current theoretical study using numerical modeling is limited by the narrow range of application of existing models. Therefore, the purpose of this thesis is to create a numerical fatigue model which takes into account the interaction of a large number of parameters affecting fatigue, includes all important aspects of fatigue crack propagation, and accurately models both the crack initiation and the stable crack growth stages.

The work has been carried out in four stages :

– *modeling of fatigue crack propagation*, where crack growth occurs through gradual damaging and failure of elements situated along the crack propagation path. The damage of the elements is calculated as a function of cyclic strain range occurring at the crack tip due to fatigue loading,
– *verification of the model*, comparing model results to experimental results, and ensuring the ability of the model to simulate parameters affecting fatigue,
– *parametric analysis of the model*, studying and ranking fatigue-related factors as a function of their importance to fatigue behavior,
– *simplification of the model*, establishing relationships between the developed model and fatigue analysis based on fracture mechanics. In addition, two examples which employ the model are also given.

The principal result of this study is the development of a model which analytically predicts the fatigue behavior of details made of structural steel, subjected to any load history, having any geometry, and containing any distribution of the fabrication-introduced residual stress. Both the crack initiation and the stable crack growth stages are considered in the model. The model is able to take into account the influence of many fatigue-related aspects, such as crack closure, small crack behavior, specimen thickness effect, fatigue threshold and fatigue behavior under cyclic compression.

The relationships established between the model and a fracture mechanics based fatigue approach enable the Paris equation constants to be determined. The identification of the most important fatigue-related factors as well as two application examples illustrate possible directions and ways for the future study of fatigue.

RÉSUMÉ

L'évaluation à la fatigue de structures existantes devient de plus en plus importante. Cependant, l'étude expérimentale du comportement à la fatigue est onéreuse, l'utilisation de courbes *S-N* fournit des résultats peu précis et les champs d'application des méthodes existantes de modélisations numériques sont trop étroits, limitant de ce fait leur utilisation. Le but de cette thèse est de créer un modèle numérique de fatigue prenant en compte un grand nombre de paramètres, incluant tous les aspects importants de l'initiation et de la propagation d'une fissure de fatigue.

Ce travail a été effectué en quatre étapes :

– *Modélisation de la propagation d'une fissure de fatigue* : dans le modèle développé, l'accroissement de la fissure a lieu par un endommagement graduel et la rupture d'éléments situés le long du chemin de la propagation. Le dommage des éléments est calculé comme une fonction de la déformation cyclique au front de la fissure, due aux sollicitations de fatigue.

– *Vérification du modèle* : le modèle développé a été comparé à des résultats expérimentaux, assurant ainsi qu'il permet de simuler les phénomènes de fatigue.
– *Analyse paramétrique du modèle* : les paramètres importants du point de vue de la fatigue ont été étudiés et classifiés en fonction de leur importance relative vis-à-vis des phénomènes de fatigue.
– *Simplification du modèle* : dans cette étape, des relations entre le modèle développé et une analyse de fatigue basée sur la mécanique de la rupture ont été établies. Deux exemples d'application du modèle sont donnés.

Le résultat principal de cette étude est le développement d'un modèle qui permet l'analyse du comportement à la fatigue de détails de construction en acier, ayant n'importe quelle géométrie et soumis à une quelconque « histoire » de sollicitations. Ce modèle permet de considérer aussi bien l'initiation de la fissure que sa propagation stable. Il peut tenir compte de l'influence d'un grand nombre d'aspects relatifs à la fatigue, comme la fermeture de la fissure, le comportement des petites fissures, l'effet de l'épaisseur, le seuil de fatigue et le comportement à la fatigue sous compression cyclique.

Les relations établies entre le modèle et une approche de la fatigue basée sur la mécanique de la rupture permettent de déterminer les constantes de la loi de Paris. L'identification des paramètres les plus importants du point de vue de la fatigue ainsi que les deux exemples d'application illustrent les directions possibles pour des études futures.

ZUSAMMENFASSUNG

Die Beurteilung des Ermüdungsverhaltens bestehender Tragwerke gewinnt zunehmend an Bedeutung. Die experimentelle Untersuchung des Ermüdungsverhaltens bestehender Tragwerke ist jedoch kostspielig, die Verwendung von Ermüdungsfestigkeitskurven (*S*-*N*-Kurven) liefert zu konservative Ergebnisse und die theoretische Analyse mittels numerischer Modelle ist aufgrund der wenigen zur Verfügung stehenden Modelle stark begrenzt. Das Ziel dieser Dissertation ist daher die Entwicklung eines numerischen Modells für Ermüdung, das die Wechselwirkung der zahlreichen ermüdungsspezifischen Einflussgrössen berücksichtigt und alle wesentlichen Aspekte der Risswachstumsproblematik einschliesst.

Die Dissertation ist wie folgt aufgebaut :
– *Modellierung des Ermüdungsrisswachstums*. Die Risse wachsen infolge zunehmender Schädigung bzw. Versagen der Elemente entlang des Risswachstumspfades. Die Schadensermittlung für die einzelnen Elemente erfolgt in Abhängigkeit der an der Rissfront auftretenden ermüdungswirksamen Dehnungen.
– *Verifizierung des Modells*. Vergleich der numerischen Simulationen mit Versuchsergebnissen. Nachweis der Eignung des Modells zur Beschreibung ermüdungsspezifischer Verhaltensweisen.
– *Parameterstudie*. Untersuchung und Klassifizierung ermüdungsspezifischer Faktoren in Abhängigkeit ihres Einflusses auf das Ermüdungsverhalten.
– *Vereinfachung des Modells*. Es wird eine Beziehung zwischen dem Modell und der bruchmechanischen Ermüdungsanalyse hergestellt. Zwei Anwendungsbeispiele des Modells werden gezeigt.

Das wesentliche Ergebnis der Dissertation ist die Entwicklung eines Modells, das die Untersuchung des Ermüdungsverhaltens von Stahlkonstruktionsdetails mit beliebiger Geometrie, mit einer beliebigen Spannungsgeschichte und mit beliebigen Eigenspannungen ermöglicht. Das Modell schliesst sowohl Rissentstehung als auch stabiles Risswachstum mit ein. Bei dem Modell können verschiedene ermüdungsspezifische Aspekte wie Rissschliessen, Verhalten bei kleinen Rissen, Einfluss der Probendicke, Risswachstumsschwelle und Verhalten bei druckerzeugenden Ermüdungslasten berücksichtigt werden.

Auf der Grundlage der zwischen Modell und bruchmechanischem Ermüdungsansatz hergestellten Beziehung können die Konstanten der Paris-Gleichung ermittelt werden. Durch die Identifizierung der wichtigsten Ermüdungsparameter sowie durch die beiden Anwendungsbeispiele werden mögliche Richtungen für zukünftige Forschung auf dem Gebiet der Ermüdung gezeigt.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to everyone who, by their support, encouragement, help or remarks have contributed to the fulfillment of this work.

Firstly, I would like to thank Professor Jean-Claude BADOUX, ex-director of ICOM and now president of the EPFL, for giving me the opportunity to work in an excellent research environment and for providing me with the opportunity to do my doctoral studies. Equal thanks go to Professor Manfred A. HIRT, present director of ICOM, for ensuring my continued employment at ICOM, but also for encouraging me to start and continue with this thesis as well as taking on the responsibility of being the thesis director. I also sincerely appreciate the many hours set aside by Dr. Peter M. KUNZ for discussions and his many useful suggestions. In particular his comments and guidance on the writing of this thesis are gratefully acknowledged.

I would also like to thank the members of the examination committee for the time and effort which they put into reading this thesis : Professor Walter GRAF of the Department of Civil Engineering, Professor John BOTSIS of the Department of Mechanical Engineering, Professor Carlo CASTIGLIONI of Politecnico di Milano, Dr. Véronique DUBOIS of Swiss Federal Aircraft Factory, and Dr. Volker ESSLINGER of Swiss Federal Laboratories for Materials Testing and Research (EMPA).

I would like to thank Dr. Pierluigi COLOMBI of the Politecnico di Milano for the very useful discussions and suggestions offered during the time he spent at ICOM in 1996 ; Dr. Roland KOLLER and Mr. Georg SOYKA of Swiss Federal Laboratories for Materials Testing and Research (EMPA) for their kind help with the preparation and execution of fatigue tests ; Dr. Simon F. BAILEY for his help with the C programming language and making some corrections to the English text ; Professor Eugene BRÜHWILER for his support during the early stages of this research ; and Mr. Gilbert PIDOUX for the preparation of test specimens.

I would also like to thank Professor Valdek KULBACH and Dr. Riho ORAS from Tallinn Technical University for making it possible for me to come to Switzerland to carry out this research.

I sincerely want to thank Mr. Robert CONNOR and Mr Laurent BIGNENS who spent many hours correcting the English text, as well as Mrs Esther von ARX and Miss Anne-Laure SCHWERZMANN who handled the administrative aspects of the thesis.

The support of all collaborators, past and present, of ICOM who contributed to this research by creating the almost ideal atmosphere for creative work are gratefully acknowledged. Special thanks to Vincent for helping me with my basic knowledge of French, but equally to Markus, Erik, Mikko, Bertrand, Jean-Marc, Alberto and Christian for their friendship. Thanks to them and many other people, my stay in Switzerland consisted of more than simply ‘extended modeling of fatigue behavior’.

I would especially like to thank my wife Helen, my daughter Ipi-Liisa and my parents for their patience in accepting my absence during the years this work has preoccupied me.

TABLE OF CONTENTS

DEFINITIONS

NOTATIONS

Latin Uppercase

A_5 elongation related to 5x diameter of specimen ;
C constant of Paris equation ;
C_1, C_2 constants of *S*-*N* curve ;
C_{el} a parameter used in the crack propagation rate equation that accounts for the fatigue threshold ;
C_m a parameter used in the crack propagation rate equation that accounts for the effect of mean stress ;
C_{pl} constant of the crack propagation rate equation ;
CTOD crack tip opening displacement ;
CTOD' cyclic crack opening displacement ;
$CTOD'_{op}$ cyclic crack tip opening displacement calculated using $K_{max,abs}$ and K_{op} ;
$CTOD_{max,abs}$ crack tip opening displacement calculated using absolute maximum stress intensity factor $K_{max,abs}$;
D damage ;
D_j damage of *j*th element, where $j=1...5$;
$D_{init,j}$ initial damage of *j*th element ;
D_{tot} total damage ;
E elastic modulus (Young modulus) ;
F_c factor that accounts for the influence of local stress distribution on the stress intensity factor K (=stress correction factor) ;
F_f factor that accounts for the influence of crack geometry *and* detail geometry on the stress intensity factor K (=geometry correction factor) ;
K stress intensity factor ;
K' cyclic strength coefficient ;
K_{eff} effective stress intensity factor ;
$K_{eff,i}$ effective stress intensity factor corresponding to the load peak i;
$K_{eff,max}$ maximum effective stress intensity factor ;
K_I stress intensity factor corresponding to the mode I crack ;
K_i stress intensity factor corresponding to the load peak i ;
K_{i^*} stress intensity factor, corresponding to the load peak i^* ;
K_{max} stress intensity factor due to maximum nominal stress ;
$K_{max,abs}$ the absolute maximum stress intensity factor among the load events, counted *from* the beginning of the load history *to* the current load event ;
$K_{max,eff}$ effective stress intensity factor corresponding to the maximum nominal load ;
$K_{max,fict}$ fictitious maximum stress intensity factor used in order to account for the effect of the mean stress on the crack initiation life ;
K_{min} stress intensity factor due to minimum nominal stress ;
$K_{min,abs}$ the absolute minimum stress intensity factor among the load events, counted *from* the beginning of the load history *to* current load event ;
$K_{min,eff}$ effective stress intensity factor corresponding to the minimum nominal load ;
K_{op} opening stress intensity factor - the stress intensity factor corresponding to the crack opening stress σ_{op} ;
K_{op}^* opening stress intensity factor used to evaluate the height of new plastic strip element ;
K_{res} stress intensity factor due to fabrication-introduced residual stresses ;

K_{tot}	overall stress intensity factor : sum of the stress intensity factors due to the nominal load and due to the fabrication-introduced residual stresses ;
L_{il}	length of the influence line ;
N	number of load cycles ;
N_D	number of load cycles corresponding to the constant-amplitude fatigue limit ;
N_{CI}	crack initiation life - number of load cycles needed to create the crack of length a_0 ;
N_{CI}*	intersection point of the double-slope $log\ \Delta\sigma_0$-$log\ N_i$-curves (Figure 6.6) ;
$N_{CI,CAeq}$	crack initiation life under the equivalent constant-amplitude load range ;
$N_{CI,VA}$	crack initiation life under variable-amplitude loading ;
N_{CI0}	crack initiation life of the 'reference case' (Chapter 5) ;
N_f	in the context of fatigue crack propagation: the number of load cycles needed to propagate a fatigue crack from *zero* size to the critical length a_{cr} ; i.e., number of load cycles until failure of the detail; in the context of *S-N* curves : fatigue resistance of the detail ;
N_{f0}	number of load cycles until failure of the 'reference case' (Chapter 5) ;
N_{fixed}	number of load cycles, fixed at certain value ;
$N_{f,1}$	number of load cycles until failure of the first element ($j=1$) ;
$N_{f,CAeq}$	fatigue life under the equivalent constant-amplitude load range ;
$N_{f,elem}$	fatigue life of element ;
$N_{f,elem,i}$	fatigue life of element corresponding to the strain range $\Delta\varepsilon_i$ and mean stress $\sigma_{m,i}$;
$N_{f,elem,k}$	fatigue life of element k ;
$N_{f,i}$	fatigue resistance of the detail at the load range $\Delta\sigma_{0,i}$;
$N_{f,VA}$	fatigue life under variable-amplitude loading ;
N_{SCG}	stable crack growth life - number of load cycles needed to propagate the crack from length a_0 to its critical (final) size a_{cr} ;
P	nominal concentrated load ;
P_{max}	nominal concentrated maximum load ;
P_{min}	nominal concentrated minimum load ;
R	ratio of the minimum and maximum nominal load (stress) ;
R	(hole) radius ;
R_0	constant in Equation (4.3) ;
R_{eff}	effective load (stress) ratio : ratio of the crack opening load (stress) to the maximum nominal load (stress) ;
$R_{eff,le}$	effective load (stress) ratio of linear-elastically behaving crack ;
R_{ys}	ratio of the maximum nominal stress to the yield stress ;
SCF	stress concentration factor ;
$SCF_{bi\text{-}linear}$	bi-linear distribution of the stress concentration factor ;
SCF_{max}	maximum value of the stress concentration factor ;
$SCF_{'reference\text{-}case'}$	distribution of the stress concentration factor for the 'reference case' in Chapter 5 ;
SCF*	an average stress concentration factor calculated over the first element at crack initiator (same as factor K_t in literature) ;
T	temperature ;
U	elastic-plastic strain energy density ;
U_{le}	strain energy density calculated using linear-elastic stress-strain analysis ;
U^*_{le}	complementary strain energy density calculated using linear-elastic stress-strain analysis ;
U^*	elastic-plastic complementary strain energy density ;
W	(half) width of the plate ;
Y	stress intensity correction factor ;

Latin Lowercase

a	(half) length of fatigue crack ;
a_0	(half) length of *initial* fatigue crack ;
a_{cr}	critical (final) (half) length of fatigue crack ;
a_{COL}	(half) length of the crack at which the *compressive* overload (COL) was applied on the detail ;
a_{TOL}	(half) length of the crack at which the *tensile* overload (TOL) was applied on the detail ;
$a_{plane\ stress}$	crack (half) length at the surface of the thick plate ;
$a_{plane\ strain}$	crack (half) length at the center of the thick plate ;
b'	fatigue strength exponent ;
c'	fatigue ductility exponent ;
d	(hole) diameter ;
dx	length of infinitely small material element ;
dy	height of infinitely small material element ;
d_i	damage caused due to *one* cycle of load range i ; damage caused to the element due to load reversal i ;
$d_{i,n}$	damage caused due to n cycles of load range i ;
$d_{j,i}$	damage caused to element j due to load reversal i ;
d_{ls}	damage due to one load spectrum ;
i	load reversal ; load peak ;
i^*	index of the tip of the elastic-plastic stress-strain hysteresis loop, where the index of another tip is i ;
$i0$	trough in the load history where starts the rain flow flowing over the peak i ;
j	jth element, using local numeration of elements (Figure 3.2) ;
k	kth element, using global numeration of elements (Figure 3.2) ;
m	exponent of Paris equation ;
m_1	the first slope of the double-slope fatigue resistance curve ;
m_2	the second slope of the double-slope fatigue resistance curve ;
m_{el}	exponent in crack propagation rate equation that accounts for the fatigue threshold ;
m_m	exponent in equation that accounts for the effect of the mean stress on crack propagation rate ;
m_{pl}	exponent in crack propagation rate equation ;
n	counter of load reversals ;
n'	cyclic strain hardening exponent ;
n_i	the number of nominal load cycles occurring at the stress range $\Delta\sigma_{0,i}$;
n_{global}	total number of elements on propagation path of fatigue crack ;
n_{local}	total number of elements used in the same time ;
n_{rev}	number of load reversals in variable-amplitude load history ;
pcf	plastic constraint factor ;
q_0	constant in Equation (4.3) ;
r	polar co-ordinate ;
r_{CI}	ratio of the crack initiation life N_{CI} to the total fatigue life N_f of the smooth specimen ;
r_{pl}	size of the monotonic plastic zone of Dugdale crack ;
$r_{pl,max,abs}$	size of the monotonic plastic zone of Dugdale crack, corresponding to the absolute maximum stress intensity factor $K_{max,abs}$;
r'_{pl}	size of the cyclic plastic zone of Dugdale crack ;
$r'_{pl,op}$	size of the cyclic plastic zone of Dugdale crack if the nominal strain range $\Delta\sigma_0 = \sigma_{max,abs}-\sigma_{op}$;
$r^*_{pl,max,abs}$	size of the monotonic plastic zone of Dugdale crack just after crack advance ;
v	velocity ;
v_{max}	opening displacement of Dugdale crack due to $\sigma_{0,max}$;

v_{op}	crack opening displacement of Dugdale crack corresponding to the crack opening stress σ_{op} ;
v_{PS}	height of the plastic strip element ;
$v_{PS,k}$	height of the plastic strip element k ;
$v_{PS,new}$	height of new plastic strip element, added at the location of failed crack tip element ;
v_{min}	opening displacement of Dugdale crack due to $\sigma_{0,min}$;
x	Cartesian co-ordinate ;
$x_{0,j}$	nearest point of element j to the crack tip ;
$x_{0,k}$	nearest point of element k to the stress concentrator ;
x_{cont}	distance between physical crack tip and the unique contact point of the crack edges ;
y	Cartesian co-ordinate ;

Greek Uppercase

ΔK	stress intensity factor range ;
ΔK_{eq}	equivalent constant-amplitude stress intensity factor range, used to normalize the influence of variable-amplitude loading ;
$\Delta K_{eff,i}$	effective stress intensity factor range, corresponding to the load reversal i ;
$\Delta K_{eff,th}$	effective threshold stress intensity factor range ;
$\Delta K_{eff}{}^*$	effective stress intensity factor range, at the intersection point of two crack propagation rate curves (Figure 6.3) ;
ΔK_{fict}	fictitious stress intensity factor range used to calculate the crack initiation life ;
$\Delta K_{fict,i}$	fictitious stress intensity factor range corresponding to load reversal i ;
ΔK_{th}	stress intensity factor range at fatigue threshold ;
ΔU_{le}	strain energy density range calculated using linear-elastic stress-strain analysis ;
ΔU	elastic-plastic strain energy density range ;
$\Delta \varepsilon$	elastic-plastic strain range ;
$\Delta \varepsilon_{el}$	elastic part of elastic-plastic strain range ;
$\Delta \varepsilon_{i}$	elastic-plastic strain range corresponding to the load reversal i ;
$\Delta \varepsilon_{le}$	strain range calculated using linear-elastic methods ;
$\Delta \varepsilon_{pl}$	plastic part of elastic-plastic strain range ;
$\Delta \sigma$	elastic-plastic stress range ;
$\Delta \sigma_{0}$	nominal stress range ;
$\Delta \sigma_{0,eff}$	effective nominal stress range ;
$\Delta \sigma_{0,i}$	ith block of the stress range in nominal stress histogram ;
$\Delta \sigma_{CUT\text{-}OFF}$	cut-off limit ;
$\Delta \sigma_{D}$	constant-amplitude fatigue limit ;
$\Delta \sigma_{e}$	equivalent constant-amplitude stress range ;
$\Delta \sigma_{el}$	elastic part of elastic-plastic stress range ;
$\Delta \sigma_{i}$	elastic-plastic stress range corresponding to the load reversal i ;
$\Delta \sigma_{le}$	stress range calculated using linear-elastic methods ;
$\Delta \sigma_{le,j}$	fatigue load of the element j - linear-elastic stress range applied on the element j ;
$\Delta \sigma_{le,k}$	fatigue load of the element k - linear-elastic stress range applied on the element k ;
$\Delta \sigma_{pl}$	plastic part of elastic-plastic stress range ;
$\Delta \sigma_{R}$	fatigue strength of the detail ;

Greek Lowercase

α	weld toe angle ;
δ	element's length ;
ε	elastic-plastic (local) strain ;

$\varepsilon_{abs,i}$	maximum elastic-plastic strain among the absolute values of strains with index *1...i* ;
ε_f	monotonic fracture ductility ;
ε_{Glinka}	elastic-plastic strain calculated using Glinka's ESED criterion ;
ε_i	elastic-plastic strain corresponding to the load peak *i* ;
ε_{i*}	elastic-plastic strain corresponding to the load peak *i** ;
ε_{i0}	elastic-plastic strain corresponding to the trough *i0* ;
ε_{le}	strain calculated using linear-elastic methods ;
ε_{max}	maximum elastic-plastic strain ;
ε_{min}	minimum elastic-plastic strain ;
ε_{Neuber}	elastic-plastic strain calculated using Neuber's rule ;
ε'_f	fatigue ductility coefficient ;
$\phi_{el,j}$, $\phi_{el,k}$	constants used in order to take into account for the simultaneous damaging of elements *j* and *k* during the crack initiation stage ;
$\phi_{pl,j}$, $\phi_{pl,k}$	constants used to account for the simultaneous damaging of elements *j* and *k* during the stable crack growth stage ;
ρ	material grain size ;
ρ	crack tip radius ;
σ	elastic-plastic (local) stress ;
σ_0	nominal stress ;
$\sigma_{0,COL}$	nominal compressive overload ;
$\sigma_{0,max}$	maximum nominal stress ;
$\sigma_{0,min}$	minimum nominal stress ;
$\sigma_{0,max,abs}$	the absolute maximum nominal stress among the load events counted from the beginning of the load history to current load event ; the absolute maximum nominal stress in the load history ;
$\sigma_{0,min,abs}$	the absolute minimum nominal stress in the load history ;
$\sigma_{0,TOL}$	nominal tensile overload ;
$\sigma_{abs,i}$	maximum elastic-plastic stress among the absolute values of stresses with index *1...i* ;
σ_{Glinka}	elastic-plastic stress calculated using Glinka's ESED criterion ;
σ_f	monotonic fracture strength ;
σ_i	elastic-plastic stress corresponding to the load peak *i* ;
$\sigma_{il,max}$	maximum stress on influence line ;
σ_{i*}	elastic-plastic stress corresponding to the load peak *i** ;
σ_{i0}	elastic-plastic stress corresponding to the load peak *i0* ;
σ_{le}	stress calculated using linear-elastic methods ;
$\sigma_{le,i}$	load of element corresponding to the load peak *i* ;
$\sigma_{le,i0}$	load of element corresponding to the load peak *i0* ;
$\sigma_{le,j}$	load of the element *j* - linear-elastic stress applied on the element *j* ;
$\sigma_{le,k}$	load of the element *k* - linear-elastic stress applied on the element *k* ;
$\sigma_{le,max}$	maximum load of element ;
σ_m	(local) mean stress : the half sum of local maximum and minimum stresses ;
$\sigma_{m,i}$	(local) mean stress corresponding to the load reversal *i* ;
σ_{max}	maximum elastic-plastic stress ;
σ_{min}	minimum elastic-plastic stress ;
σ_{Neuber}	elastic-plastic stress calculated using Neuber's rule ;
σ_{op}	crack opening stress - a minimum nominal stress which causes a complete opening of the crack ;
σ_{res}	fabrication-introduced residual stress ;
σ_x	component of linear-elastic stress acting in the direction of *x*-axis ;
σ_y	component of linear-elastic stress, acting in the direction of *y*-axis ;
σ_{ys}	monotonic yield stress (corresponding to the 0.2% strain) ;
σ'_f	fatigue strength coefficient ;

σ'_{ys}	cyclic yield stress ;
τ_{xy}	linear-elastic shear stress, acting in the direction of *x*- and *y*-axis ;
ξ	normalized co-ordinate ;
θ	angle from the crack plane ;

Diverse

$A(B)$	*A* as function of *B* ;
da/dN	fatigue crack propagation rate ;
$(da/dN)_{th}$	threshold fatigue crack propagation rate (about 10^{-8} [mm/cycle]) ;
$f_{i,j}(\theta)$	function of θ ;
$g(\xi)$	function of crack opening profile ;
$Max(A ;B)$	maximum of *A* or *B* ;
$Min(A ;B)$	minimum of *A* or *B* ;
$N_f(SN)$	fatigue life calculated using *S*-*N* curves ;
$N_f(MF)$	fatigue life calculated using 'model F' ;
$\sigma_{i,j}$	stress components acting on the infinitely small material element.

TERMINOLOGY

fatigue crack	a crack appearing at a crack initiator due to fatigue loading. It is assumed that such cracks propagate along a path perpendicular to maximum principal tensile stresses (mode I fatigue crack) ;
crack closure effect	reduction in stress intensity range at the crack tip due to premature closure of the crack edges. Premature closure means that crack becomes partially closed even if nominal tensile stress is applied on the detail ;
crack initiation	the first stage of fatigue crack propagation ;
crack initiator	stress concentrator at which fatigue crack initiation occurs ;
crack propagation	growth of the fatigue crack due to fatigue loading ;
crack stable growth	the second stage of fatigue crack propagation ;
critical crack size	length of the fatigue crack at which ductile or brittle fracture of the detail can take place ;
cycle	a load history between two adjacent maximum or minimum stress peaks ;
cycle counting	determination of the stress ranges from the load history ;
cyclic plastic zone	part of the plastic zone which undergoes *cyclic* plastic deformations ;
fatigue threshold	arrest of fatigue damage evolution ;
load history	load variation in time ;
load-life relationship	relationship between load-related parameter and fatigue life ;
load response of elements	change in elastic-plastic stress and strain of an element due to change in loading of element ;
load sequence	order of the load peaks in load history ;
material memory effect	in a periodical complex history, after the cyclic 'stabilization' of the material, larger hysteresis loops always enclose smaller ones. If

	a small strain excursion occurs inside a larger strain excursion, the larger hysteresis loop is not affected by the smaller one and the hysteresis history after the closure of the smaller loop behaves as if the smaller loop had never happened. In other words the material ‘remembers’ the previous loading path ;
nominal load	far-field load (stress, concentrated force etc.), applied on the edges of structural detail ;
opening stress	minimum nominal stress, causing complete opening of the fatigue crack, where word ‘complete’ refers to the situation that there is no contact between crack edges ;
opening stress intensity factor	stress intensity factor calculated using opening stress ;
plastic zone	region of plastified material around stress concentrators created during loading of the detail ;
plastic strip	region of plastified material at the edges of fatigue crack ;
reversal	a load history between adjacent maximum and minimum stress peaks
stress amplitude	one half of the stress range ;
stress concentrator	regions where the geometry of the detail changes or/and where material structure changes. Stress concentrators can be holes, welds, material defects etc. ;
stress concentration factor	ratio of the local linear-elastic stress to the nominal stress ;
stress range	difference of maximum and minimum stress ;
stress spectrum	stress range histogram obtained by cycle counting ;
structural detail	some locally defined area of overall structure (for example : joints, parts of the structure containing welded attachments, etc.) ;

ABBREVIATIONS

AW	as welded ;
CA	constant-amplitude ;
CI	crack initiation ;
COL	compressive overload ;
CTOD	crack tip opening displacement ;
ESED	equivalent strain energy density ;
FCP	fatigue crack propagation ;
HT	heat treated ;
LEFM	linear elastic fracture mechanics ;
MF	‘model F’ ;
NP	needle peened ;
SCG	stable crack growth ;
SIF	stress intensity factor ;
SN	*S-N* curve ;
TOL	tensile overload ;
UCG	unstable crack growth ;
VA	variable-amplitude ;